

# An effective local routing strategy on the BA network

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## Abstract

In this paper, We propose a effective routing strategy on the basis of the so-called nearest neighbor search strategy by introducing a preferential delivering exponent  $\alpha$ . we assume that the handling capacity of one vertex is proportional to its degree when the degree is smaller than a cut-off value  $K$ , and is infinite otherwise. It is found that by tuning the parameter  $\alpha$ , the scale-free network capacity measured by the order parameter is considerably enhanced compared to the normal nearest-neighbor strategy. Traffic dynamics both near and far away from the critical generating rate  $R_c$  are discussed. We also investigate  $R_c$  as functions of  $m$ (connectivity density),  $K$ (cutoff value). Due to the low cost of acquiring nearest-neighbor information and the strongly improved network capacity, our strategy may be useful and reasonable for the protocol designing of modern communication networks.

*Key words:* complex networks, scale-free, local routing strategy

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## 1 Introduction

A variety of systems in nature can be described by complex networks and the most important statistical features of complex networks are the small-world effect and scale-free property[1,2,3]. It may serve as a very useful tool for understanding nature and our society. Since the discovery of some common interesting features of many real networks such as small-world phenomena by Strogatz and Watts[1] and Scale-free phenomena by Albert and Barabási [2], processes of dynamics conducting on the network structure such as traffic congestion of information now have drew more and more attention from engineering and physical field

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[12,13,14,15,16,17,18,19,20,21,22], due to the importance of large communication networks such as WWW[4] and Internet [5] in modern society. Evolution of networks structure and interplay of traffic dynamics also play an important role in the research of the traffic system including Internet, highway network and so on , they want to understand and explain the process of dynamics on the underlying structure.

Many previous excellent works focus on the evolution of structure driven by the increment of traffic[6,7,8] and some explore how different topologies impact the traffic dynamics[12,13]. Some works[10,11] gave several models to mimic the traffic routing on complex networks by introducing randomly selected source as well as particles (packets) generating rate and destination of each particles[18,19,20]. Those models define the capacity of networks described by critical generating rate, too. At this critical rate, a continuous phase transition from free flow state to congested state occurs. In the free state, the numbers of created and delivered particles are balanced, leading to a steady state. While on the jammed state, the number of accumulated particles increases with time due to the limited delivering capacity or finite queue length of each vertex.

We believe that the study on the network search is very important for traffic systems, for the existence of particles routing from origin to destination and communication cost is very meaningful. a few previous studies incorporate the search strategies and the traffic processes on networks[9,10,11,20]. In this paper, we present a traffic model in which particles are routed only based on local topological information with a single tunable parameter  $\alpha$ . In order to maximize the nodes handling and delivering capacity of the networks which can be measured by an introduced order parameter  $\eta$ , the optimal  $\alpha$  is found out. We also check the dynamic properties in the steady state for different  $\alpha$  including average number of particles versus vertex degree, particles distribution and particles traveling time distribution. The dynamics right after the critical generating rate  $R_c$  exhibits some interesting properties independent of  $\alpha$ , which indicates that although the system enters the jammed state, it possesses partial capacity for forwarding particles. Our model can be considered as a preferential walk among neighbor vertexes. We arrange the paper as follows. In the following section we describe the model in detail, in Sec.3 simulation results of traffic dynamics are provided in both the steady and congested states, A conclusion and discussion are given in the last section.

## 2 Model

our traffic model is described as follows: at each time step, there are  $R$  particles generated in the system, with randomly chosen sources and destinations, and all vertexes can deliver at most  $C$  particles toward their destinations, which is one of the most interesting properties of the whole traffic network. The capacity of each

vertex is set to be proportional to its degree when the degree is smaller than a cut-off value  $K$ , and be infinite otherwise, We choose  $C_i = k_i$ . As a remark, there is difference between the capacity of network and vertexes. The capacity of the whole network is measured by the critical generating rate  $R_c$  at which a continuous phase transition will occur from free state to congestion. The free state refers to the balance between created particles and removed particles at the same time. When the system enters the jam state, it means particles continuously accumulate in the system and At last few particles can reach their destinations. In order to describe the critical point accurately, we use the order parameter[9,10,11,20]:

$$\eta(R) = \lim_{t \rightarrow \infty} \frac{C_i < \Delta N_p >}{R \Delta t} \quad (1)$$

$\Delta N_p = N(t + \Delta t) - N(t)$  with  $< \dots >$  indicates average over time windows of width  $\Delta t$  and  $N_p(t)$  represents the number of data particles within the networks at time  $t$ , here. For  $R < R_c$ ,  $< \Delta N > = 0$  and  $\eta = 0$ , indicating that the system is in the free state with no traffic congestion. Otherwise for  $R > R_c$ ,  $\eta \rightarrow r$ , where  $r$  is a constant larger than zero, the system will collapse at last. To navigate particles, each vertex performs a local search among its neighbors. If a particle's destination is found in the searched area, it will be delivered directly to its target, otherwise, it will be forwarded to a neighbor  $j$  of vertex  $i$  according to the probability:

$$\Pi_{i \rightarrow j} = \frac{k_j^\alpha}{\sum_l k_l^\alpha} \quad (2)$$

here, the sum runs over the neighbors of vertex  $i$  on the searched area and  $\alpha$  is an adjustable parameter studied by us in the next context. Once a particle reaches its destination, it will be canceled from the system. As shown in Fig.1, the order parameter versus generating rate  $R$  by choosing different value of parameter  $\alpha$  is displayed. It is easy to Find that the capacity of the system is not alike for different  $\alpha$ , thus, a natural question is addressed: what is the optimal value of  $\alpha$  for making the network's capacity maximal in our model?

Many studies[1,2,3] indicate that many communication networks such as *Internet* are not homogeneous like random or regular networks. Barabási and Albert proposed a famous model (*BA* for short) called scale-free networks[2], of which the degree distribution is in good accordance with modern communication networks, which has a power law distribution  $P(k) \propto k^{-\gamma}$ . Our study is based on the so-called *BA* network, we construct the network structure following the same method used in Ref.[2]: starting from  $m$  fully connected vertexes, a new vertex with  $m$  links is added to the existing graph at each time step according to the rule of preferential attachment i.e. the probability of being connected to an existing vertex is proportional to the degree of that vertex. Here, we choose  $m = 5$  and network size  $N = 1000$  fixed for simulations.

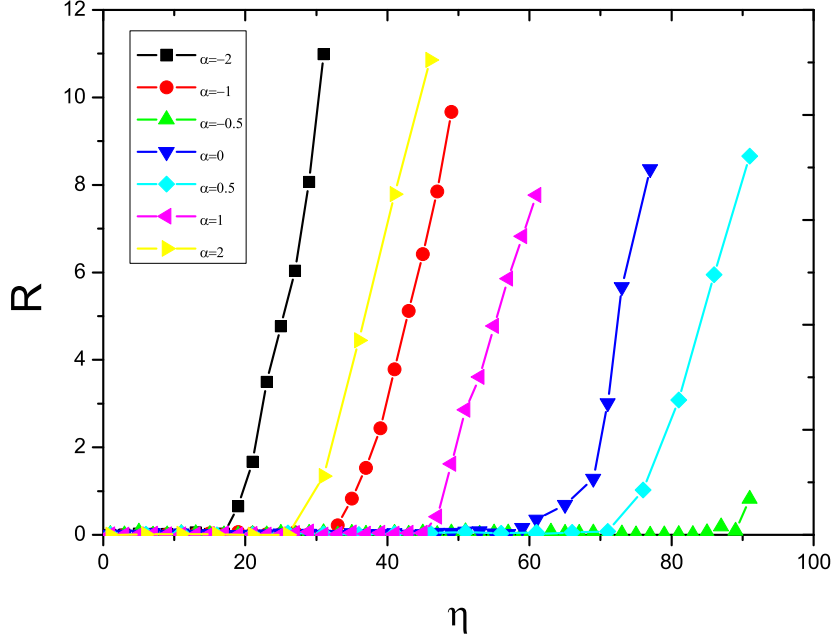


Fig. 1. (color online). The order parameter  $\eta$  versus  $R$  for BA network with different free parameter  $\alpha$ . Other parameters are networks size  $N = 1000$ ,  $K = 1000$ , and  $m = 5$ .

We should also note that the queue length of each vertex is assumed to be unlimited and the *FIFO* (First in First out) discipline is applied at each queue[9,10,20,?]. Another important rule called path iteration avoidance (*PIA*) is that a link between any pair of vertexes is not allowed to be visited more than twice by the same particle[9,10,11]. Without this rule, the capacity of the network is quite low due to many times' unnecessary visiting to the same links by the same particles, which does not exist in the real traffic systems. We note that this *PIA* routing algorithm does not damage the advantage of local routing strategy. If each particle records the links it has visited, the *PIA* can be easily performed. One can find that this rule does not need the global topological information. Therefore, we think this rule is rational and can considerably improve the network capacity.

With the development of science and technology, the handling capacity of the main central node can be set up large enough artificially, that is why we propose the cutoff value  $K$ . We hold that it is more practical and more reasonable.

### 3 Simulations

We have carried on the simulation under the definition of the model, the order parameter  $\eta$  versus generating rate  $R$  with choosing different value of parameter  $\alpha$  is

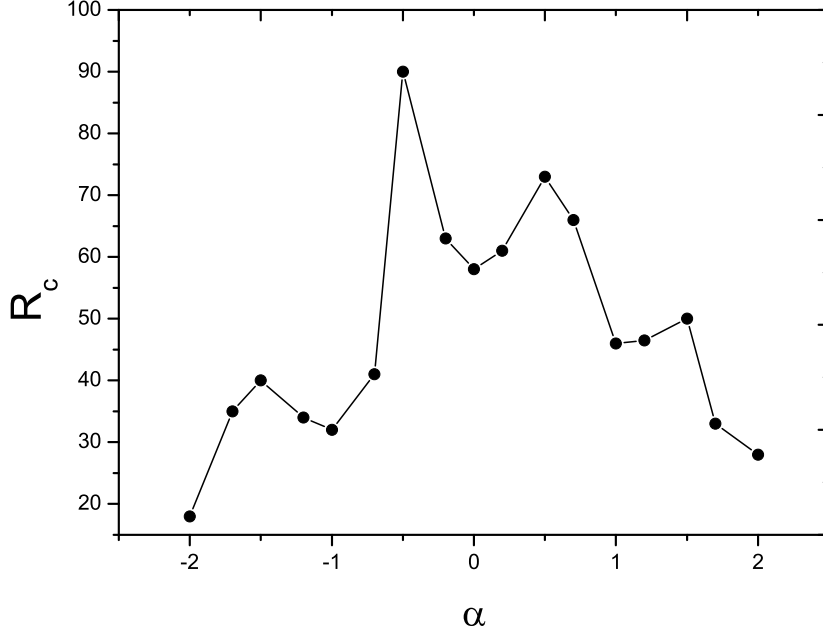


Fig. 2. The critical  $R_c$  versus  $\alpha$  with network size  $N = 1000, K = 1000$ . The maximum of  $R_c$  corresponds to  $\alpha = -0.5$  marked by a black solid line.

reported, As shown in Fig. 1. One can see that, for all different  $\alpha$ ,  $\eta$  is approximately zero when  $R$  is small; it suddenly increases when  $R$  is larger than the critical point  $R_c$ . It is easy to find that the capacity of the system is not the same for different  $\alpha$ . For the same  $\eta$ , when  $\alpha = -0.5$ , the  $R$  reaches its max. We can preliminarily determine that it is the best situation.

We also observed the handling capacity  $R_c$  for different  $\alpha$  in one system, one can read from Fig.2 that the tolerance is the best when  $\alpha = -0.5$ . This is another strong evidence to show that  $\alpha = -0.5$  is a perfect point. One can read an multi-peak effect at  $\alpha = -1.5, -0.5, 0.5$  and  $1.5$ . This is an interesting phenomena, which we also gets in Fig.5. The perfect point is  $\alpha = 0.5$  in that case, however, The nature of symmetry about network can be reflected.

We have studied the critical point  $R_c$  affected by the cutoff value  $K$ , As shown in Fig.3. When the  $K$  increases, the capacity of BA network measured by  $R_c$  considerably has the optimal performance at  $K = 200$ . Although  $K$  is a variable parameter, the system always reaches its best case at  $\alpha = -0.5$ . Furthermore, we study the critical point  $R_c$  affected by the link density of BA network. As shown in Fig.4, increment of  $m$  considerably enhances the capacity of BA network measured by  $R_c$  due to the fact that with high link density, particles can more easily find their target vertexes. One can read the Fig.3 and Fig.4, then know that the critical  $R_c$  reaches its max when  $\alpha = -0.5$  at the same  $K$  or  $m$ .

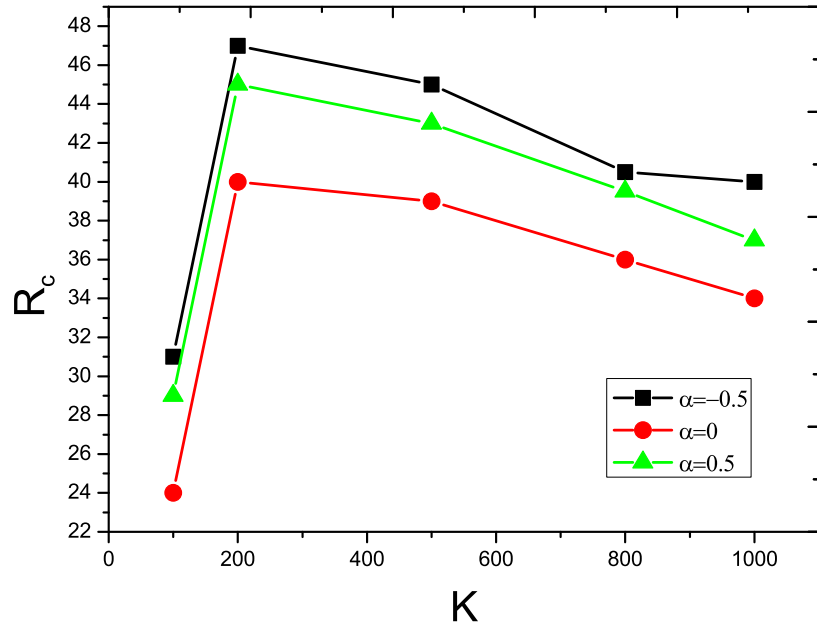


Fig. 3. (color online). The variance of  $R_c$  with the increasing of  $K$

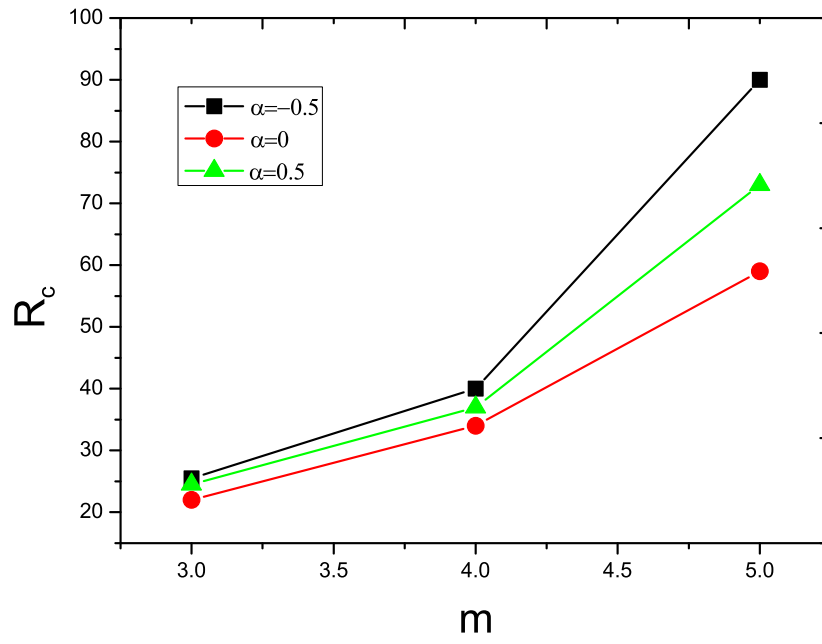


Fig. 4. (color online). The variance of  $R_c$  with the increasing of  $m$

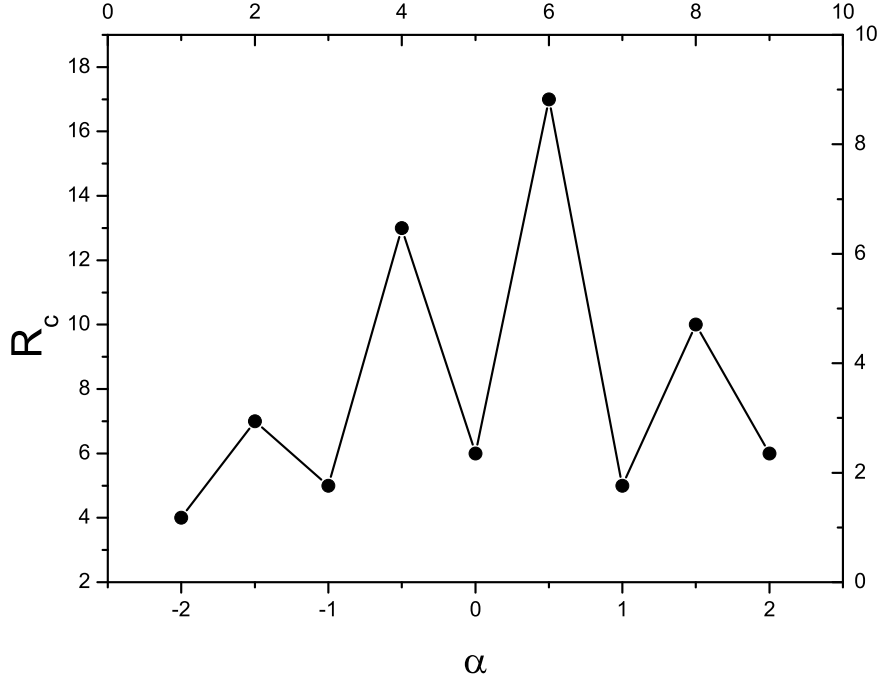


Fig. 5. The critical  $R_c$  versus  $\alpha$  for  $C_i = C \times k^2$  and  $C = 1$ . here, the network size  $N = 5000$

As shown in Fig.5 above, we have simulated the case when  $C_i = C \times k^2$  and we find that the system has the optimal performance at  $\alpha = 0.5$ . This simulation is based on the practice that the capacity of the network is stronger with the development of technology.

Simulation results demonstrate that the optimal performance of the system corresponds to  $\alpha = -0.5$ . Compared to previous work by Kim *etal.* [18], one of the best strategies is *PRF* (preferential choice, which means the vertex with the larger degree has the higher probability to receive particles) corresponding to our strategy with  $\alpha = -0.5$ . By adopting this strategy a particle can reach its target vertex most rapidly without considering the capacity of the network. This result may be very useful for search engine such as *google*, but for traffic systems the factor of traffic jam cannot be ignored. Actually, average time for the particles spending on the network can be also reflected by the system capacity. It will indeed reduce the network's capacity if particles spend too much time before arriving at their destinations.

## 4 Conclusion

We have introduced a new routing strategy only based on local information, trying to give a solution to the problem of traffic congestion in modern communication networks. Influenced by two factors of each node's capacity and the cutoff value  $K$ , the optimal parameter  $\alpha = -0.5$  is obtained with maximizing the whole system's capacity. In addition, the property that scale-free network with occurrence of congestion still possesses partial delivering ability suggests that only improving processing ability of the minority of heavily congested vertexes can obviously enhance the capacity of the system. The variance of critical value  $R_c$  with the increment of  $m$  and  $K$  is also discussed. Our study may be useful for designing communication protocols for large scale-free communication networks due to the local information the strategy only based on and the simplicity for application. The results of current work may also shed some light on alleviating the congestion of modern technological networks. Further work could be carried out, for the queue length of each vertex is infinite.

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